

Multidimensional Region Connection Calculus

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Abstract

The best way to support commonsense reasoning over geographic data is via qualitative spatial reasoning over spatial objects and their relations. The Region Connection Calculus (RCC) (Randell et al. 1992) family is one of the well-known logical languages for formalizing topological relationships that describe commonsense spatial knowledge. In this paper, we modify and extend RCC-8 to propose a topological model accepting multi-dimension geometric features as an input. We will compare the model with the 9-Intersection Model (9-IM) in our future work to show that our model offers more flexibility for geographic information systems (GIS).

1. Introduction

One of the most promising research areas in GIS focuses on the development of reasoning formalisms that exploit qualitative (symbolic) representations (Montello 2001)(Raffaeta et al. 2003). This type of reasoning is also compatible with human common-sense understanding (Falomir 2016). The main aspects of qualitative spatial knowledge representation and reasoning are topology, distance, shape and orientation (Dylla et al. 2016), and among them topology is the most fundamental aspect, as it can only make qualitative distinctions (Cohn & Hazarika 2001). While topology supports the ability to say whether two objects can be continuous with each other, or something can be inside, outside, abutting, or surrounding something else (Varzi 1994); mereology (parthood theories) facilitates reasoning about parts of objects (Van Harmelen et al. 2008). Thus, the most studied aspect of the qualitative spatial theories has a mereotopological approach in which the *parthood* and *contact* relations play the main roles and other spatial relations are derived from them.

Modeling of mereotopological relations has applications in robotic navigation (Kunze et al. 2014)(Falomir et al. 2013), GIS (Fogliaroni 2013)(Al-Salman 2014) and high level vision (Cohn et al. 2006). In the literature, two main models have been introduced to find (mereo)topological relationships between two objects (mostly applied in GIS): the 9-Intersection Model (9-IM) (Egenhofer & Franzosa 1991) and the Region Connection Calculus (RCC) (Randell et al. 1992). They have completely different approaches for distinguishing spatial relationships. The former has a mathematical approach rooted in point-set topology, while the latter has a logical approach.

In this paper, we review these two main methods in section 2, with the focus on how they deal with dimensionality. We then provide the definitions of geometric objects relevant to the discussion in section 3, extracted from the Standards for Geographic Information (OGC 2013 and ISO 2012). We then propose our model, called multi-dimensional RCC, in section 4. Section 5 presents our plan to compare the expressive power of multi-dimensional RCC's relations in comparison to 9-IM as well as a discussion of our current and future work. The sum up of this paper is provided in this Section 6.

2. Current approaches

In this section, we review 9-IM and RCC as the two main approaches, because our method is based on the latter, while the former is the applied method in current GIS.

2.1 9-Intersection Model

The 9-Intersection Model (9-IM) finds mereotopological relations between two features by using a comparison matrix. Existence of the intersections is registered in the matrix entries. In other words, 9-IM uses the *interior* (\circ), *boundary* (∂) and *complement* ($'$) topological operations along with the intersection operation from set-theory to

define all the topological relationships. Consider the following matrix for features X and Y as an example:

$$\begin{array}{ccc} X^\circ \cap Y^\circ & X^\circ \cap \partial Y & X^\circ \cap Y' \\ \partial X \cap Y^\circ & \partial X \cap \partial Y & \partial X \cap Y' \\ X' \cap Y^\circ & X' \cap \partial Y & X' \cap Y' \end{array}$$

The entries of the matrix show whether the intersection between two parts of (extracted by topological operators) the objects is empty or not. In order to achieve the necessary interpretation of the topological relations of 9-IM, we have to think of a feature as lying in a space (R^n) which has the same dimension as the feature. As a contradictory example, consider S as a line (2D object) in three-dimensional space (in which its co-dimension¹ is 1). If it is modeled as a subset of this space with usual topology, mathematically its whole is considered to be the boundary of the line ($\partial S = S$) and it does not have any interior ($S^\circ = \emptyset$). However, in 9-IM, the boundary of S is considered to be its two end-points, and its interior ($S \setminus \partial S$) is an interval between these two end points and disjoint from them. Hence, it means that S is interpreted as a one-dimensional subset of space instead of a three-dimensional subset of it. Current GIS commonly adopt the 9-IM model of spatial relations. 9-IM provides $2^9 = 512$ relations but not all of them are possible in the real world. So, a subset of possible relations is adopted by the OGC (OGC 1999), consisting of eight topological relations between two regions in two-dimensional space.

2.2 Region Connection Calculus

The Region Connection Calculus family is a first-order logical formalism. Regions are spatial primitives in it and a binary connection relation, which is interpreted as ‘the topological closures of regions x and y have at least one point in common’ [page442-(Hirtle & Frank 1997)], is its non-logical primitive:

$$C(x, y) = cl(x) \cap cl(y) \neq \emptyset$$

in which ‘cl’ returns the closure of a region. This relation has reflexivity and symmetry properties and a set of relations are derived from it (see **Table 1**) named RCC. A subset of these relations has the Jointly Exhaustive and Pairwise Disjoint (JEPD) property which can distinguish eight mereotopological relations between two simple regions in two-dimensional space. These relations are the same as the relations defined by 9-IM and have been named “RCC-8” (JEPD relations {PO, EQ, TPP, NTPP, TPPi, NTPPi, EC, DC}).

¹ Co-dimension shows a dimensional difference between the space and the object.

Table 1 RCC relations

Relation	Definition	Interpretation
DC (x,y)	$\neg C(x, y)$	x is disconnected from y
P (x,y)	$\forall z[C(z, x) \rightarrow C(z, y)]$	x is part of y
Pi (x,y)	$P(y, x)$	y is part x
PP (x,y)	$P(x, y) \wedge \neg P(y, x)$	x is proper part y
PPi (x,y)	$PP(y, x)$	y is proper part x
O (x,y)	$\exists z[P(z, x) \wedge P(z, y)]$	x overlaps y
DR (x,y)	$\neg O(x, y)$	x is discrete from y
EC (x,y)	$C(x, y) \wedge \neg O(x, y)$	x is extensionally connected to y
TPP (x,y)	$PP(x, y) \wedge \exists z(EC(z, x) \wedge EC(z, y))$	x is tangential part of y
TPPi (x,y)	$TPP(y, x)$	y is tangential part of x
NTPP (x,y)	$PP(x, y) \wedge \neg \exists z(EC(z, x) \wedge EC(z, y))$	x is non-tangential part of y
NTPPi (x,y)	$NTPP(y, x)$	y is non-tangential part of x
EQ (x,y)	$P(x, y) \wedge P(y, x)$	x is equal to y
PO (x,y)	$O(x, y) \wedge \neg P(x, y) \wedge \neg P(y, x)$	x is partially overlaps y

Moreover, a set of quasi-Boolean operators supports RCC-8. They are: *sum*(x,y) ‘which reads as’ the sum of x and y’, *compl*(x) which reads as ‘the complement of x’, *prod*(x,y) which reads as ‘the product (intersection) of x and y’ and *diff*(x,y) ‘which reads as’ the difference of x and y (the part of x that does not overlap y)’ (Cohn et al. 1997).

A brief comparison of the 9-IM and RCC-8 methods is shown in **Table 2**.

Table 2 Comparison of 9-IM & RCC-8

9-IM	RCC-8
Whole-based approach	Decomposing approach
Mathematics-based	Logic-based
Primitive spatial entity is point	Primitive spatial entity is region
Representing topological relations between objects with co-dimension ≥ 0	Representing topological relations between objects with co-dimension=0
High computational cost	Lower computational cost
Composition table for regions only	Complete composition table available
More expressive	Less expressive

2.3 Dimensionality in 9-IM and RCC-8

Among various important qualitative concepts in QSR, handling dimensionality is crucial in GIS applications. Sometimes we need the notion of lower-dimensional entities in three-dimensional space. For instance the representation of a bridge linking two pieces of land over a river may consist of both lines and polygons and require queries to be performed that can deal with mixed dimensions. In the following two paragraphs, we will evaluate these properties in the 9-IM and the RCC-8 frameworks.

We discuss the dimensionality of 9-IM and RCC-8 using some figures. Consider two situations presented in **Figure 1(a&b)**. 9-IM's matrix (as shown below) is the same for these two situations:

$$\begin{array}{ccc} \emptyset & \neg\emptyset & \neg\emptyset \\ \emptyset & \emptyset & \neg\emptyset \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \end{array}$$

This reveals that the 9-IM alone is insufficient to determine the dimensionality of the intersection (Galton 2004). Consideration of the dimension of the intersection part in 2D space was addressed in Clementini's work (dimensionally extended-9IM or DE-9IM (Clementini et al. 1993)). The OGC standard (OGC 2010) used DE-9IM as a tool to return mereotopological relations by considering the dimension of the intersection. Although it is more practical than 9-IM, it is not able to deal with complex objects (with hole(s) and multi parts) which the logical structure of RCC-8 can handle more easily. So, our model is based on the RCC's concepts.

Different dimensionality in RCC-8 is controlled by an important axiom. This axiom excludes regions with co-dimension greater than zero. The axiom is $\forall x\exists y[NTPP(y,x)]$. In other words, this axiom does not find relations between a line and a region in 2D space. For instance, consider the case presented in **Figure 1.a**, L_1 and L_2 are two lines in R^2 such that L_2 is a proper part of L_1 without touching L_1 's endpoints. According to the definition of NTPP relations (see **Table 1**), there must be no region that is connected to both lines simultaneously. However, the region R is exactly such a region for the lines in R^2 . It means that there is no non-tangential proper parts for L_1 , which violates the axiom. Although there is a possibility of interpreting this situation in a way that considers the common part as a point, such an interpretation completely removes the notion of external connection and

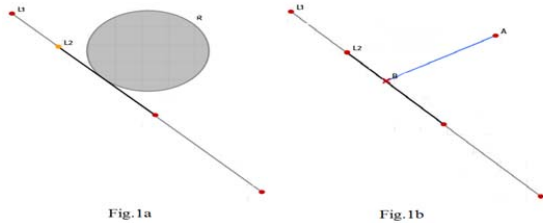


Figure 1 Dimensionality of the intersection

consequently TPP and NTPP. Our model will try to provide a facility to describe this situation and similar cases.

3. Definition of geometric features

According to the OGC standards (ISO 19125-2 2004; OGC 2010), real world phenomena are stored as features. They are mostly represented as 0, 1 and 2 dimensional geometries and can be simple or complex. It is assumed that features are topologically closed (OGC 2010) (non-empty) which means the boundary of the feature belongs to it. We will use this concept in the model to be presented in the next section. In the following, we briefly recall the definitions of features from OGC Standard (OGC 2010)(ISO 19125-2 2004) which form the foundation of our model.

Def.1. The interior of a feature (x°) is a set of positions that are in the feature (x) but which are not on its boundary. Topologically, it is the union of all open sets contained in the feature.

Def.2. The closure of a feature (\bar{x}) is the intersection of all closed sets containing the feature (x).

Def.3. The boundary of a feature (∂x) represents the limit of an entity. Topologically, it is a point-set difference between the feature's closure and interior ($\bar{x} \setminus x^\circ$).

Def.4. The exterior of a feature (x^-) is the difference between the universe and the closure, or $R^n \setminus \bar{x}$.

Def.5. A simple point is a zero-dimensional topological feature, representing a position and it does not have any boundary.

Def.6. A complex point is a collection of separate simple points, and it does not have any boundary.

Def.7. A simple line is a one-dimensional topological feature, representing a single, continuous and homogeneous one-to-one mapping of the unit interval in the plane. It has start and end points that show the direction of the line and its boundary.

If the mapping function does not have the one-to-one property, it represents a line with self-intersection. A ring is a particular case of a line with self-intersection and it does not have any boundary.

If a one-dimensional topological feature does not have the continuous property, it represents as a collection of several mappings of an interval and its name is a complex line.

Def.8. A surface is a two-dimensional topological feature representing a single, connected interior connected exterior and homogeneous one-to-one mapping to a region on the plane.

The boundary of a surface is the set of oriented, closed curves that delineate the limits of the surface.

If a surface does not have the interior connection property, we obtain a region which consists of multi parts. Also, removing the constraint of exterior connection will intro-

duce a region with a hole(s). In the OGC simple features specification (OGC 2010), surfaces with/without holes are implemented as polygons.

Def.9. A complex surface is a surface (region) with a hole(s) and/or consisting of multi-parts.

To develop efficient and scalable techniques for storage and querying 3D objects, we also need to have the definitions of the 3D objects. As mentioned before, almost all of the current GIS use 2D representation and analysis, but there are some assumptions and definitions in the OGC standards to describe 3D objects (Gröger et al. 2012). In these definitions, the z coordinate of a point typically represents altitude or elevation.

Def.10. A solid is a closed surface in 3D space which has an interior that is separated from the rest of the space by that surface. The boundary of the solid is a surface, all of whose facets are polygons.

4. Definition of multi-dimensional RCC

As mentioned before, RCC-8 can only find topological relations between regions of dimension R^n (for $n>0$) in which the value of n is the same for both regions. Moreover, the OGC standard (OGC 2010; ISO 19125-2 2004) has a cross relation in the list of topological relations which apply to point/line, point/surface, line/line and line/surface situations on the plane (see **Figure 2**). Contrary to the remaining topological relationships, this relation must also deal with geometries of mixed dimensions. In addition, the relations between point/solid, line/solid, surface/surface and surface/solid are not definable in 3D space via RCC (because of its limitations). So, we have to modify its spatial primitive to provide the capability to describe these situations. The spatial primitives must be generic spatial features instead of specifically regions. In other words, the approach has to be developed for a multi-dimensional space.

As a result, the crosses relation must accept solid, surface and line geometry types. One way to achieve this goal is by defining the boundary of the spatial entities. In geometry, the dimension of the boundary is one dimension less than the spatial entity, which means that two end points are the boundary of a line, a curve is the boundary of a surface and a collection of polygons is the boundary of a solid.

This goal will be achieved by adopting object definitions represented in the OGC specification (OGC 1999) as well as modifying RCC's spatial relations based on these definitions. We take the universe of discourse to be closed solids (possibly disconnected), closed polygons (also possibly disconnected), closed lines (possibly disconnected as well) and sets of isolated points. The *connection* relation in this

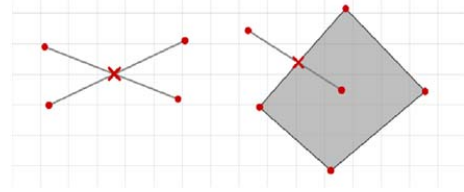


Figure 2 Cross relation from OGC Standard (OGC 2010)

model has the same definition and properties as RCC-8 and the other relations are derived from it.

The disconnected relation is the same as RCC-8:

Def.11. $DC(x,y) =_{def} \neg C(x,y)$

Also, the *part* and *proper part* relation relations in this model have the same definition as the RCC-8:

Def.12. $P(x,y) =_{def} \forall z [C(z,x) \rightarrow C(z,y)]$

Def.13. $PP(x,y) =_{def} P(x,y) \wedge \neg P(y,x)$

The *equal* relation showing the identicality between two regions is:

Def.14. $EQ(x,y) =_{def} P(x,y) \wedge P(y,x)$

In the original RCC, the *externally connected* and *overlap* relations are used to refine the *connection* relation by considering the third region in the common part. However, as discussed in Section 2.3, the common part could be point, line or even surface in three-dimensional space. To overcome the problem, we have to redefine these relations (*EC* & *O*). The re-definition needs to consider a new topological primitive: the *Boundary* relation.

The *Boundary* relation $B(x_1, x)$ returns the boundary (x_1) of the feature (x). Intuitively, the boundary of an object is a part of the object (not its interior) which is in contact with its exterior. The dimension value of x_1 ranges over $\{0, 1, 2, Null\}$. Now, the boundary primitive is defined as follows in which $compl(x)$ ¹ is the complement of x .

Def.15.²

$$B(x_1, x) =_{def} PP(x_1, x) \wedge C(x_1, compl(x))$$

Although this definition requires that x_1 is both a proper part of x and connected to the complement of x , which may appear to be contradictory, the definition of the connection relation ensures that this is valid, because the connection relation refers to the closure of x_1 and the closure of the complement of x , not of x_1 and the complement of x themselves.

¹ $compl(x) =_{def} \forall y [\forall z [C(z, y) \rightarrow \neg P(z, x)]]$

² B is a relation and if x is a closed ring its boundary is empty which means there is no value x_1 for that B is true. Similarly it is not true for a point.

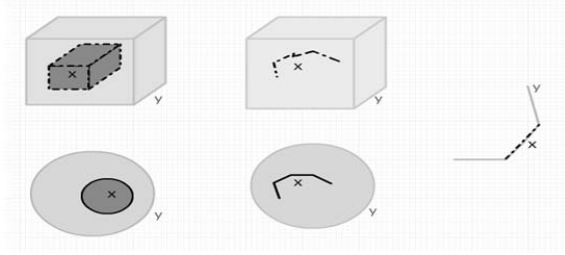


Figure 3 NTPP relation

Moreover, this definition is not the same as the definition of boundary in mathematical topology in which the boundary of an object coincides with its closure\interior.

Hence, we redefine the rest of necessary relations based on this primitive. The *non-tangential proper part* relation is defined firstly, because EC and O relations are derived from it.

Def.16.

$$NTPP(x, y) =_{def} PP(x, y) \wedge \forall x_1 \forall y_1 [B(x_1, x) \wedge B(y_1, y)$$

$$\rightarrow DC(y_1, x_1)]$$

Figure 3 illustrates the definition of *NTPP*. As you can see there is no difference between the represented semantics here and the original RCC for regions.

The redefinition of *TPP* relation is:

$$\mathbf{Def.17.} \quad TPP(x, y) =_{def} PP(x, y) \wedge \neg NTPP(x, y)$$

Now, we can introduce the *overlap* relation which has a different definition in comparison to the original RCC:

$$\mathbf{Def.18.} \quad O(x, y) =_{def} \exists z [NTPP(z, x) \wedge NTPP(z, y)]$$

This definition mentions that the common part is a non-tangential proper part of both features. Also, it provides a chance to define a *cross* relation (see **Def.21**).

In the same way as for the original RCC, the *partially overlap* relation is a refinement of the *overlap* relation to exclude *proper part* situations (see **Figure 4**):

$$\mathbf{Def.19.} \quad PO(x, y) =_{def} \exists t TPP(t, x) \wedge TPP(t, y)$$

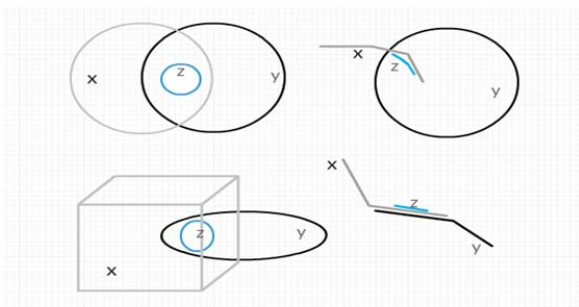


Figure 4 PO relation

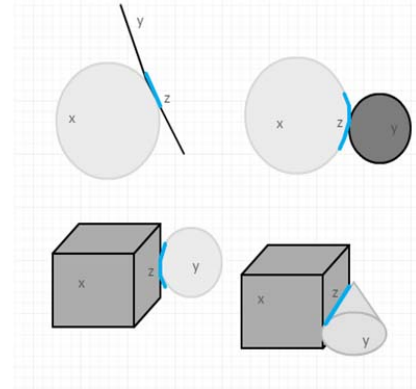


Figure 5 EC relation

Two more types of *connection* relations that the *overlap* relation cannot encapsulate, can be defined in this model. This is the result of accepting different features as an input. They are the *externally connected* and *cross* relations. The *externally connected* relation is defined as:

Def.20.

$$EC(x, y) =_{def} C(x, y) \wedge \neg O(x, y) \wedge [\forall t [\forall x_1 \forall y_1 B(x_1, x) \wedge B(y_1, y)$$

$$\rightarrow PP(t, x_1) \vee PP(t, y_1)]]$$

The illustration of the *EC* relation is shown in **Figure 5**. The common part *t* must be a part of the boundary of one of the features (this is achieved through the existence quantifier $\exists t$). This relation has the same semantics as the original RCC.

Finally, the *Boundary* relation helps us to introduce a completely new topological relation, *cross* relation:

Def.21.

$$CR(x, y) =_{def} O(x, y) \wedge \neg PO(x, y) \wedge \neg P(x, y)$$

This model let us detect various cross situations¹ some of which are presented in **Figure 6**. The most important difference between the PO and CR relations is their common part. In the PO relation, the dimension of the common part is always equal to the lowest dimensional participant in the relation, while it can have any dimension in CR relation.

The inverse relations of the asymmetric *part* and *proper part* relations are as below:

$$\mathbf{Def.22.} \quad Pi(x, y) =_{def} P(y, x)$$

$$\mathbf{Def.23.} \quad PPi(x, y) =_{def} PP(y, x)$$

$$\mathbf{Def.24.} \quad NTPPi(x, y) =_{def} NTPP(y, x)$$

$$\mathbf{Def.25.} \quad TPPi(x, y) =_{def} TPP(y, x)$$

¹ Note that the cross relation is between objects from any dimension except two 3D objects that they form a kind of "fat cross"; this is not an instance of the cross relation, but just of the PO relation.

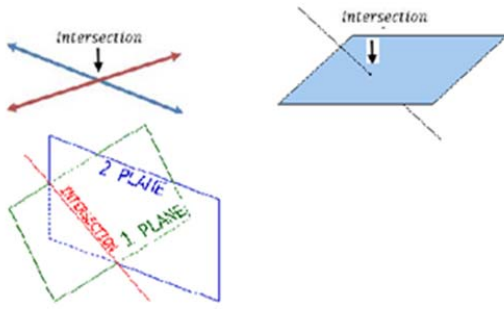


Figure 6 CR relation

The definition of the last relation to make this model as complete as the original RCC is the *discrete* relation:

Def.26. $DR(x,y) = \text{def } EC(x,y) \vee DC(x,y)$

The JEPD property of the set of relations {DC, EC, PO, TPP, NTPP, TPPi, NTPPi, EQ, and CR} is plausible. The complete set of relations {DR, DC, EC, P, PP, PO, TPP, NTPP, Pi, PPi, TPPi, NTPPi, EQ, and CR} can be enclosed in a relational lattice, which is given in **Figure 7**. The ordering of the represented relations can be achieved directly from the definitions.

In the next section, we provide a brief overview of the spatial reasoning system that will be developed.

5. Future work

Our model will be completed with an appropriate spatial reasoning approach. Moreover, to show its functionality we will compare it with 9-IM.

5.1 Spatial reasoning

For some purposes it is enough to have a representation of spatial knowledge, but what makes intelligent systems intelligent is their ability to reason about given knowledge (Van Harmelen et al. 2008). Multi-dimensional RCC is defined in first order predicate calculus in terms of the spatial primitive, the *connection* relation. Although reasoning in this calculus is undecidable, we plan to define a composition table for multi-dimensional RCC as a reasoning mechanism. The composition table will include the funda-

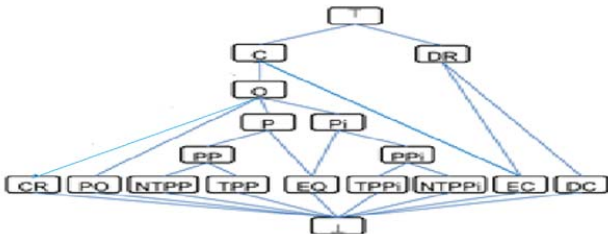


Figure 7 Lattice of Multi-dimensional model

mental rules to accomplish qualitative spatial reasoning. The algebraic composition operator (\circ) is defined between two relations, r_1 and r_2 , and the composition relation is:

$$r_3(x, z) = r_1 \circ r_2 = \{(x, z) \mid \exists y : (x, y) \in r_1, (y, z) \in r_2\}$$

The composition table will present the conceivable outcomes of compositions for each combination of relations. The outcomes are the disjunction of the basic relations. The composition table of multi-dimensional RCC can be achieved by extending RCC-8's composition table. To overcome the difficulty of proving the composition table, we can take two approaches: (1) showing that each disjunction in each cell is necessary; (2) showing that there are no missing disjunctions (Clementini & Cohn 2014).

5.2 Comparison with 9-IM

To show the effectiveness of multi-dimensional RCC, we will try to find correspondences with 9-IM (and even DE-9IM) as a system currently implemented in the OGC standard. To achieve this goal, we will use the `Relate` function, which is available in the OGC standards, to return the value of a 9-IM's matrix in the form of a row. The value's range is over {T (True), F(False), *(Don't care)}. The 'T' shows the non-empty intersection, while 'F' shows the empty one, e.g. `Relate(x, y, "FF*FF**")`. Also, this function specializes to the dimensionality values of the intersection part based on the DE-9IM definitions if its initial output is 'T'. Finding the equivalent expression of the relations without referring to the dimension of the intersection (from DE-9IM) makes the multi-dimensional RCC more practical.

6 Conclusion

There has been a strong interest in multi-dimensional mer-eotopology (Hahmann 2011) in fields such as geology and archaeology. Our contribution, multi-dimensional RCC, is an approach to tackle this issue. It is defined by adding a new primitive $B(x,y)$ to define the boundary of the feature that leads us to have a chance to increase the expressive power of the model by introducing a new spatial relation, the cross relation, $CR(x,y)$. The primitive also modifies some the original RCC definitions. The spatial primitive is not just limited to regions (as in the original RCC), but instead it accepts features from various dimensions ($\{0, 1, 2, 3\}$) embedded in R^3 , and it also handles complex objects.

As we mentioned in the previous sections, we plan to develop the composition table of multi-dimension RCC as well as finding equivalent expressions between it and DE-9IM. Moreover, the model must be developed by describing the common part(s) in more detail, to improve its ability to expressively describe a wider range of possible rela-

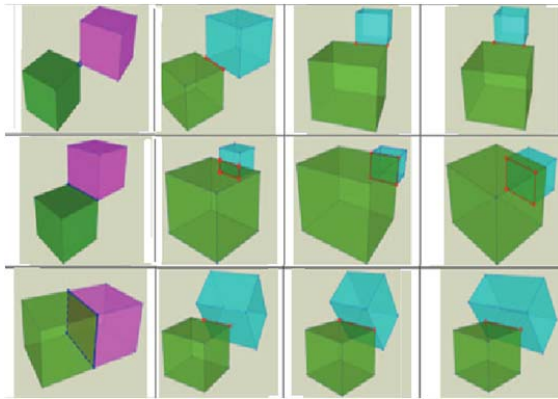


Figure 8 More details relations

tions in multi-dimensions, including the examples presented in **Figure 8**.

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