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The Logic of Occurrence

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Abstract

A general problem in qualitative physics is determining the consequences of assumptions about the behavior of a system. If the space of behaviors is represented by an envisionment, many such consequences can be represented by pruning states from the envisionment. This paper provides a formal *logic of occurrence* which justifies the algorithms involved and provides a language for relating specific histories to envisionments. The concepts and axioms are general enough to be applicable to any system of qualitative physics. We further propose the concept of *transverse quantities* as a general solution to qualitative versions of Zeno's paradox. The utility of these ideas is illustrated by a rational reconstruction of the pruning algorithms used in FROB, a working AI program.

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1 Introduction

A goal of qualitative physics is to predict the behavior of physical systems. One technique, *envisioning*, generates all possible behaviors of a system, relative to a particular set of background assumptions. Informally, any specific behavior of a system (a *history*) corresponds to a path through the system's envisionment, and vice versa. This correspondence is essential to using envisionments. For example, interpreting measurements can be viewed as constructing a correspondence between a (usually partial) history and an envisionment. However, the correspondence between histories and envisionments has never been adequately formalized.

Formalizing this relationship has two benefits. First, it provides additional grounding for theories of measurement interpretation and diagnosis (such as [7,8,4]) Second, we use the formalization to generalize existing domain-specific algorithms for inferring the consequences of assumptions about behavior. The problem is this: Any assumptions we make about a system's behavior (or any additional information we obtain concerning it) restricts its possible behavior in the future. For example, in engineering design we might assume that a boiler's rupture pressure is never reached or that the water level inside it never goes above a particular height. Alternately, if we are observing a moving object then we might be able to calculate its initial energy, and thus place bounds on its location. In each of these cases a particular subset of behavior is directly ruled out, but other behaviors are indirectly ruled out as well. Understanding these indirect consequences is useful for determining if we have imposed the correct constraints on our design or if our assumptions about an observed system are correct. Figure 1 shows an example of these conclusions drawn by an existing AI program, `FROB` [5], that reasons about moving point-masses in a 2D world. The formalization presented here explicitly identifies the intuitions embodied in these algorithms, making them available for general application.

The following section describes the logic of occurrence, introducing *registrations* to represent the relationship between envisionments and histories. The axioms for inferring additional consequences of behavioral assumptions are also presented. These concepts are sufficiently general to apply to any system of qualitative physics. Next, we describe a form of Zeno's paradox which plagues qualitative physics, and propose *transverse quantities* as a general, domain-independent solution. Section 4 illustrates the utility of this logic by a rational reconstruction of `FROB`'s algorithms. Section 5 discusses further implications and plans for extensions.

2 Theory

We begin by introducing formal definitions for certain aspects of envisionments, qualitative states, and histories. Next, we describe the concept of a *registration*, a mapping between an envisionment and a history. We then describe the relationships between occurrences of states, including axioms which can be used to derive the consequences of assuming that some states must (or must not) occur.

Figure 1: An example of reasoning about occurrence

Part (a) shows a typical FROB scenario. Given a diagram, FROB computes a qualitative description of free space. The user places a ball in the position shown and gives it an initial velocity. Part (b) (left) shows FROB's initial envisionment, based only on assuming that the ball is in SRO, and on the right is the result of pruning the envisionment to reflect the consequences of various additional assumptions. Each arrow and circle depicts a potential qualitative state of the ball (see [5] for details). The additional assumptions are shown in FROB's answers below. The only quantitative information used was the initial position and velocity of the ball in using energy constraints to calculate the maximum height the ball could reach. All other conclusions are based solely on qualitative information. A query session with FROB has been hand-translated into an English "dialog" for clarity.

(a)

(b)

Q: Why can't the ball leave the diagram going to the right?

A: Because you assumed the ball would pass through S31 going left and up.

Q: Why can't the ball leave the diagram out the top?

A: Energy.

Q: Why can't the ball reach S11?

A: Because you assumed it could not pass through S41.

2.1 Preliminaries

Definition 1 (Envisionment) *An* envisionment $\mathcal{E}$ *represents all possible qualitative states a particular system may take on and all legal transitions between them. The function* $States : \mathcal{E} \longrightarrow \{states\}$ *maps from envisionments to the set of states it contains.*

For generality we assume as little as possible about the nature of qualitative states. We assume that each qualitative state can be described by a finite amount of information. Ergo, qualitative states may be tested for identity with finite computational effort[1]. Furthermore, we assume that all states in an envisionment are fully defined with respect to whatever physics and domain model was used to generate them. In other words, the descriptions of states do not contain unbound variables. Aside from the ability to tell if two states are the same, for our present purposes the only properties we need to talk about are the transitions between them.

Definition 2 (Transition functions) *The functions* $Befores : s \longrightarrow \{states\}$, $Afters : s \longrightarrow \{states\}$ *describe all transitions involving a state s. State* $s_i \in Befores(s)$ *exactly when the envisionment contains a transition from* s_i *to* s. *State* $s_i \in Afters(s)$ *exactly when the envisionment contains a transition from* s *to* s_i.

Two special cases surface repeatedly when thinking about envisionments. The first are states whose $Befores$ is empty, what in automata theory are called "Garden of Eden" states. The second are states whose $Afters$ are empty, which represent states the system remains in forever.

Definition 3 (Eden states) *A state is* Eden *exactly when* $Befores(S_i) = \{\}$.

Definition 4 (Final states) *A state* S_i *is* Final *exactly when* $Afters(S_i) = \{\}$.

An envisionment compactly represents all possible histories (up to some resolution). If the envisionment is correct, every possible history of the system corresponds to some path through the envisionment and vice versa. Paths are defined to be a continuous chain of behaviors through the envisionment:

Definition 5 (Paths) *A* Path *is a sequence of states* $s_1, \ldots, s_n$ *such that* $s_{i+1} \in Afters(s_i)$. *Single states are also paths. The functions PathStates, PathStart, and PathEnd map from a path to the set of qualitative states which comprise it, the first state, and the final state, respectively. The term* $paths(\mathcal{E})$ *denotes all the paths in the envisionment.*

That is all we need to say about envisionments. For histories we assume the notational machinery in [9,6]. Specifically, we assume a history $\mathcal{H}$ is composed of episodes and events which are temporally extended and spatially bounded. Except

[1] This rules out elements of $\Re$ as constituents of a qualitative state.

for determining state identity, we will ignore durations and hence usually just speak of histories as being composed of episodes (we assume the function *Episodes* maps from a history to the set of episodes which comprise it). Since our logic is designed to specify the reasoning of a finite agent, we assume that histories are generated, either by a physics or by observation, and are finite. The function *FinalEpisode* maps from a history to the temporally last episode generated so far.

Unlike state descriptions in envisionments, we allow partial descriptions of episodes to model reasoning with incomplete data. We need a way to say that an envisionment state (which is generic) is the description of a particular episode of a history. To maintain generality we must take the matching operation as a primitive, and introduce a primitive to express its results:

Definition 6 (ConsistentWith) *$ConsistentWith(e, s)$ is true whenever episode e describes behavior of the system that is not inconsistent with state s.*

If in some application more information can be gathered about the physical system, then conclusions of *ConsistentWith* can be non-monotonic. We relegate these control issues to programming, where they belong. We further define the set of *PInterps*, the possible interpretation of an episode e, to be the set of states s such that $ConsistentWith(e, s)$ is true. *PInterps* are central to the ATMI theory of measurement interpretation; see [7,8] for details.

2.2 Registrations

Intuitively, a registration is a mapping which relates a history to an envisionment by identifying episodes in the history with corresponding states in the envisionment. The concept of a registration has been used informally in most work in qualitative physics ([3,6,10,13], but has never been adequately formalized. To define registrations we must first be able to say that an episode represents the occurrence of a state:

Definition 7 (OccursAt) *Given $s \in States(\mathcal{E})$, $e \in Episodes(\mathcal{H})$, $OccursAt(s, e)$ is true exactly when the state s represents what is happening during e.*

Notice that *OccursAt* is stronger than *ConsistentWith*, since a partial specification of e will allow it to match several states, even though only one state (or path of states, see below) is what actually occurs. The next axiom enforces this intuition:

Axiom 1 (OccursAt Consistency)

$$\forall s \in States(\mathcal{E}) \forall e \in Episodes(\mathcal{H})[OccursAt(s, e) \Rightarrow ConsistentWith(e, s)]$$

Collections of *OccursAt* statements are the building blocks of registrations.

Definition 8 (Registration) *A registration $\mathcal{R}$ of a history $\mathcal{H}$ with envisionment $\mathcal{E}$ consists of a set of OccursAt statements (denoted $Occurrences(\mathcal{R})$).*

The next axioms ensure that registrations correspond to paths through the envisionment. Allowing partially specified episodes means that a path of qualitative states may occur within a given episode, since the properties which distinguish the individual states may not be apparent from the history. Another complication arises if we allow the history to have temporal gaps. In the simplest case where two episodes *meet*, the accuracy condition is:

Axiom 2 (Neighbor accuracy)

$$
\begin{aligned}
&\forall e_1, e_2 \in Episodes(\mathcal{H})\ [Meets(e_1, e_2) \Rightarrow \\
&\quad \exists p_1, p_2 \in paths(\mathcal{E})\ s.t.\ [\forall s_i \in PathStates(p_1)[OccursAt(s_i, e_1) \in Occurrences(\mathcal{R})] \\
&\qquad \wedge \forall s_j \in PathStates(p_2)[OccursAt(s_j, e_2) \in Occurrences(\mathcal{R})] \\
&\qquad \wedge \forall s_j \in States(\mathcal{E})[OccursAt(s_j, e_1) \in Occurrences(\mathcal{R}) \Rightarrow s_j \in PathStates(p_1)] \\
&\qquad \wedge \forall s_j \in States(\mathcal{E})[OccursAt(s_j, e_2) \in Occurrences(\mathcal{R}) \Rightarrow s_j \in PathStates(p_2)] \\
&\qquad \wedge PathEnd(p_1) \in Befores(PathStart(p_2))]
\end{aligned}
$$

Accuracy across gaps simply means that there is some path through the envisionment that serves as a "bridge" between the (possibly singleton) paths which represent the episodes before and after the gap:

Axiom 3 (Gap accuracy)

$$
\begin{aligned}
&\forall e_1,\ e_2 \in Episodes(\mathcal{H})\ [\neg \exists e_3 \in Episodes(\mathcal{H}) s.t. Between(e_3, e_1, e_2) \Rightarrow \\
&\quad \exists p_1, p_2 \in paths(\mathcal{E})\ s.t.\ [\forall s_i \in PathStates(p_1)[OccursAt(s_i, e_1) \in Occurrences(\mathcal{R})] \\
&\qquad \wedge \forall s_j \in PathStates(p_2)[OccursAt(s_j, e_2) \in Occurrences(\mathcal{R})] \\
&\qquad \wedge \forall s_j \in States(\mathcal{E})[OccursAt(s_j, e_1) \in Occurrences(\mathcal{R}) \Rightarrow s_j \in PathStates(p_1)] \\
&\qquad \wedge \forall s_j \in States(\mathcal{E})[OccursAt(s_j, e_2) \in Occurrences(\mathcal{R}) \Rightarrow s_j \in PathStates(p_2)] \\
&\qquad \wedge [\exists p_3 \in paths(\mathcal{E})[PathEnd(p_1) = PathStart(p_3) \wedge PathStart(p_2) = PathEnd(p_3)]]]
\end{aligned}
$$

The concepts introduced up to this point suffice to provide the grounding for theories of measurement interpretation and diagnosis which must relate histories to envisionments. Next we examine how to use registrations to generate expectations about the future.

2.3 Constraining the future

To an agent observing a system, a registration represents what has gone on so far. Knowing what has happened constrains the future. Other constraints on the future include quantitative information and assumptions, as mentioned above. Since the envisionment already contains all states and transitions possible without these assumptions, their only effect will be to exclude other states from occurring. Essentially, the envisionment is "pruned" to reflect the fewer models possible with the additional information.

First we need a bridge from what we know about the history to the future. The function *InitialStates* maps from registrations to the set of envisionment states which are the end of the known history.

Axiom 4 (Definition of InitialStates)

$$\forall s \in States(\mathcal{E})\ [s \in InitialStates(\mathcal{R}) \Leftrightarrow OccursAt(s, FinalEpisode(\mathcal{H})) \in Occurrences(\mathcal{R})]$$

We require at least one starting place, but allow more than one to allow reasoning with partial information:

Axiom 5 (InitialStates Existence)

$$|\, InitialStates(\mathcal{R})\, | \geq 1.$$

2.4 Expressing Occurrence assumptions

Now we must provide a vocabulary for expressing the consequences of external constraints on states in a form that can be used to constrain envisionments. Several forms of external constraints have already been explored in qualitative physics, including explicit user assumptions, consequences of additional data (e. g. numerical simulations, energy constraints in [5]), and measurements [7,8]).

No matter what the source of data, a few simple distinctions suffice for our purposes:

Axiom 6 (Status of states) *Consider a registration $\mathcal{R}$ involving envisionment $\mathcal{E}$. For every state $s \in States(\mathcal{E})$, exactly one of the following is true:*

$$Possible(s, \mathcal{R}), \quad Required(s, \mathcal{R}), \quad Excluded(s, \mathcal{R})$$

Intuitively, $Possible(s, \mathcal{R})$ means that s represents a behavior which the system may undergo during some episode of the history. $Excluded(s, \mathcal{R})$ means that the behavior represented by s will never occur in the history, and $Required(s, \mathcal{R})$ means that the behavior represented by s must occur.[2] We now add to each registration a (possibly empty) set of *Excluded* and *Required* statements, called its *ConstraintSet*. All that remains is to specify the consequences of these occurrence assumptions.

2.5 Consequences of Occurrence assumptions

A central fact of occurrence is that to be in some state, you either have to start there or get there. We need to refine the notion of paths to include the notion of a *legal* path:

Axiom 7 (Path legality) *A path p is* legal *w. r. t. $\mathcal{R}$ when no state on it is excluded, i. e.,*

$$Legal(p, \mathcal{R}) \Leftrightarrow \neg Excluded(s_1, \mathcal{R}) \wedge \ldots \wedge \neg Excluded(s_n, \mathcal{R})$$

[2] It is easy to extend this logic to include *Required* and *Excluded* for transitions as well.

One state is *attainable from* another when there is a legal path of non-excluded states between them.

Axiom 8 (Attainability)

$$AttainableFrom(s_1, s_2) \Leftrightarrow \exists p \in paths(\mathcal{E})\ s.t.\ [PathStart(p) = s_2 \wedge PathEnd(p) = s_1 \wedge Legal(p, \mathcal{R})]$$

Now we can express the intuition we were aiming at:

Axiom 9 (Reachability)

$$\forall s \in states(\mathcal{E})\ [[\neg \exists s_i \in InitialStates(\mathcal{R}) s.t.\ AttainableFrom(s, s_i)] \Rightarrow Excluded(s, \mathcal{R})]$$

This axiom expresses what we mean, but a local version is often useful in algorithms. A trivial consequence of this axiom is that a non-Eden state is excluded unless at least one of its $Befores$ is not excluded (otherwise, it cannot participate in any legal non-singleton path, much less one from an initial state).

Axiom 10 (*NoPast* law)

$$\forall s \in \mathcal{R}\ [[\neg Eden(s) \wedge s \ni InitialStates(\mathcal{R})] \Rightarrow [\forall s_i \in Befores(s)[Excluded(s_i, \mathcal{R})]] \Rightarrow Excluded(s, \mathcal{R})]$$

Importantly, if there are cycles in the envisionment – and there usually are – then the local version is incomplete (see Section 4).

Attainability is a fairly natural concept. Just as important, but less intuitive, is the idea that behaviors must have consistent continuations into the future. The slipperiness of this idea comes from two sources. First, while all histories have an initial state (or states) by declaration, some behaviors can potentially go on forever (such as oscillations). This means we cannot simply use attainability of final states as a means of pruning, for we would be ruling out legitimate behaviors. Second, we must be careful to rule out qualitative versions of Zeno's Paradox. This issue will be detailed in the next section. Here we focus on the principle of *good continuation.*

Good continuation means that, unless the state is a final state, the behavior it describes will end at some time. When it does, one of the behaviors in its $Afters$ must represent the behavior which occurs next (since $\mathcal{E}$ is assumed to be complete). If every state in $Afters$ is already excluded, then that state cannot occur. The next axiom captures this intuition that, if further episodes are possible, then one of those possibilities must occur.

Axiom 11 (*NoFuture* law) *Unless s is a final state,*

$$[\forall s_i \in Afters(s) Excluded(s, \mathcal{R})] \Rightarrow Excluded(s, \mathcal{R})$$

Consider an agent reasoning about a history $\mathcal{H}$ being observed in real time. An agent with finite capabilities will only know a finite portion of the object's history. If the agent had a total envisionment describing the possible behaviors (or constructed the relevant parts on demand), using the ATMI theory to construct a registration will provide an explanation of the history. Furthermore, any states which have been excluded from the envisionment (or fragment thereof) by the axioms above cannot ever appear in any extension of the current history. This gives the agent a set of expectations: If any excluded behavior shows up in future observations, then either the constraints imposed on the behavior, the observations, the domain model, or some combination of these, are wrong.

Consider the projection $\mathcal{E}_{\mathcal{H}}$ of $\mathcal{E}$ to be the subset of states and transitions of $\mathcal{E}$ which are not excluded under $\mathcal{H}$, (i.e., a transition is included only when the states before and after are not excluded). Clearly $|\mathcal{E}_{\mathcal{H}}| \leq |\mathcal{E}|$, since as $\mathcal{H}$ grows there are more constraints on $\mathcal{E}$. The possibility of equality arises because some of the constraints may be redundant.

3 The Qualitative Zeno's Paradox

A problem lurks in the concept of continuation introduced above. The *NoFuture* law, like Axiom 10, is local. It turns out that local laws laws are insufficient to capture our intuitions about good continuation of action. Essentially, programs using the logic described so far fall prey to a form of Zeno's Paradox. Consider the situation in Figure 2a. A ball is bouncing up and down, heading left. We assume the ball never reaches the wall, and that the ball is perfectly elastic so that it doesn't stop. Any reasoning engine based on the logic of occurrence presented so far will think this kind of behavior is perfectly reasonable, even though it violates common sense. The problem is not peculiar to motion: Consider now Figure 2b. Suppose there is friction between the block and the table. Then the amount of energy each cycle will be less, and eventually it will stop[3]. But if we exclude the possiblity of the block stopping from the envisionment, the *NoFuture* law will not detect a problem because each state in the cycle has another element of the cycle in its *Afters*! Some oscillations can last forever (at least ideally), so we cannot simply define the problem away. A non-local technique must be used to avoid the Qualitative Zeno's Paradox (hereafter QZP).

To express the pruning conditions we must introduce an abstraction that will cover both the leftward motion of the ball and the energy of the mass-spring combination:

Definition 9 (Transverse quantities) *A* transverse quantity $\mathcal{T}$ *with respect to a cycle* $\mathcal{C}$ *in* $\mathcal{E}$ *is a property of the envisioned physical system such that:*

[3]See [6] for how this can be proven in QP theory. Advocates of classical continuity in qualitative physics (e.g. [2,13] will claim that the block indeed never stops. We prefer here to model the common intuition, since classical continuity leads directly to Zeno's paradox.

Figure 2: The Qualitative Zeno's Paradox

Reasoning about occurrences is subject to a form of Zeno's paradox. Consider these scenarios:

(a) A perfectly elastic ball is bouncing up and down on a horizontal surface, heading towards the wall on the left. Assume it cannot reach the wall.

(b) Friction is acting between the sliding block and the table. Assume that the combination never stops.

In both cases the assumption at the end makes the behavior intuitively implausible. Yet the logic presented so far will not see these as problematic.

(a)

(b)

1. $\mathcal{T}$ *exists for every state in* $\mathcal{C}$.
2. $\mathcal{T}$ *has a constant upper (alternately: lower) bound.*
3. $\mathcal{T}$ *is increasing (alternately: decreasing) over the cycle.*

If a $\mathcal{T}$ can be identified for some cycle $\mathcal{C}$, then we can avoid Zeno's paradox by forbidding $\mathcal{C}$ whenever all states corresponding to leaving the cycle (some of which will represent $\mathcal{T}$ reaching its limit point) are excluded. Let $Exits(\mathcal{C})$ be the set of exit states (i.e., the union of the $Afters$ for all $s \in \mathcal{C}$ minus the cycle states themselves).

Axiom 12 ($\overline{\text{Zeno}}$) *For all cycles* $\mathcal{C}$ *with a transverse quantity,*

$$[\forall s_i \in Exits(\mathcal{C})\ Excluded(s_i, \mathcal{R})] \Rightarrow [\forall s_j \in \mathcal{C} Excluded(s_j, \mathcal{R})]$$

Careful application of this law avoids QZP. The next section illustrates.

4 Reconstructing FROB

Here we show how FROB's ability to reason about occurrences, illustrated in Figure 1, can be understood in terms of these ideas. Given the history $\mathcal{H}$ known so far and a set of constraints concerning a ball's possible histories, FROB finds what possibilities are ruled out by these assumptions. In FROB's domain there are four types of behavioral constraints:

Requirements: The user can assume certain states must occur, or a ball must be in particular places, sometime in its future. As $\mathcal{H}$ evolves the set of requirements may shrink, since some new episode may satisfy a required state.

Exclusions: The user can assume that certain states must not occur, or that a ball cannot ever be in particular places. Excluding a place causes all states including that location to be excluded.

Elasticity: If a ball is assumed perfectly elastic then states corresponding to the ball stopping are excluded. If the ball is assumed perfectly inelastic then states corresponding to the ball flying away from a surface after a bounce are excluded.

Energy: Given a quantitative position and velocity, FROB calculates the maximum height it could reach. If a place is completely above this height, then all states including that location are excluded.

Figure 3: FROB's algorithm for reasoning about occurrences

1. Generate new attainable envisionment $\mathcal{E}$ from *FinalEpisode*($\mathcal{H}$).
2. For all states s, mark *Possible*.
3. For each s directly excluded by constraints, mark *Excluded*.
4. Until no further states are pruned,
   4.1 Use local pruning rules (Figure 4).
   4.2 Prune unreachable states (Figure 5).
   4.3 Prune QZP cycles.
   4.4 If initial state is excluded, signal error.

Figure 3 outlines the algorithm. In FROB's domain envisioning is cheap, so a new envisionment is computed which just contains those states attainable from the end of the current $\mathcal{H}$. Initially all states are assumed possible (step 2). The direct consequences of the behavioral constraints are added, and then pruning algorithms are applied repeatedly until no more states are excluded.

The algorithms used in pruning are much like those used for "garbage collection" in computer languages which allow dynamic allocation of memory. Step 4.1 provides local pruning via the NoFuture and NoPast laws to quickly get rid of states which are obviously ruled out (see Figure 4). This step is essentially a "reference count" garbage collector, using the number of possible *befores* and *afters* to determine whether a state is to be excluded. Just as reference count algorithms fail to work with circular storage structures, this step will fail to detect cycles which are excluded. Step 4.2 applies reachability (axiom 9) to remove these cycles (see Figure 5). This algorithm also keeps track of what required states are accessible through each state, so that states which do not allow all required states to occur can be ruled out. Step 4.4. enforces axiom 5.

To apply the $\overline{\text{Zeno}}$ axiom, FROB identifies places where such oscillations might occur along with the corresponding transverse quantity. If the oscillation is UP/DOWN the transverse quantity is motion in the LEFT or RIGHT direction, and if the oscillation is LEFT/RIGHT the transverse quantity is motion in the UP or DOWN direction. The cycle in each place involving a transverse quantity[4] is examined in step 4.3 to see if either (a) the ball may stop, (b) it can leave the place, or (c) if the transverse quantity can reverse direction (i.e. bouncing around inside a sealed box. If any of these are possible the cycle is okay, and otherwise it is pruned to avoid QZP.

[4]Not every cycle is subject to QZP – a perfectly elastic ball bouncing straight up and down on a horizontal surface will bounce forever.

Figure 4: Using local laws to prune states

This algorithm uses the NoFuture and NoPast laws to prune states without good continuations and those which cannot be reached from the initial state(s). Cycles are handled seperately.

1. Let $Q = \{states(\mathcal{E})\}$.
2. Until Q empty,
   - 2.1 $s = Pop(Q)$
   - 2.2 If $Excluded(s, \mathcal{R}) \vee Final(s)$, ignore.
     Otherwise, if every $s_i \in Afters(s)$ is $Excluded$,
     - 2.2.1. Mark s as $Excluded$.
     - 2.2.2. For every $s_i \in Befores(s)$,
       If $s_i \in Q \vee Excluded(s_i, \mathcal{R})$ then ignore.
       Otherwise, $Push(s_i, Q)$.
   - 2.3 If $Excluded(s, \mathcal{R}) \vee s \in InitialStates(\mathcal{R})$, ignore.
     Otherwise, if every $s_i \in Befores(s)$ is $Excluded$,
     - 2.3.1. Mark s $Excluded$.
     - 2.3.2. For every $s_i \in Afters(s)$,
       If $s_i \in Q \vee Excluded(s_i, \mathcal{R})$ then ignore.
       Otherwise, $Push(s_i, Q)$.

Figure 5: Finding unreachable states and imposing requirements

This algorithm is complicated by the need to exclude those states which do not allow every required states to be reached. Call the set of required states R. Each state s will have an associated set R_s, representing the required states found before or after it on some path, and a bit *mark* to indicate whether or not the state was reached on the search (in case $R = \{\}$).

1. For each $s \in InitialStates(\mathcal{R})$, $MarkRequired(s, \{\}, \{\})$.
2. For each $s \in states(\mathcal{E})$,
   2.1 If $mark(s) = 0$ mark s *Excluded*.
   2.2 If $R_s \neq R$, mark *Excluded*,
   with reason being $R - R_s$.

Procedure $MarkRequired(s, path, sofar)$

1. If s *Excluded*, ignore.
2. $mark(s) \leftarrow 1$
3. If $s \in R$,
   3.1 For every $s_i \in path$, $R_{s_i} \leftarrow s \cup R_{s_i}$
   3.2 $sofar \leftarrow s \cup sofar$.
4. $R_s \leftarrow sofar \cup R_s$
5. For every $s_i \in Afters(s)$,
   $MarkRequired(s_i, s \cup path, sofar)$

5 Discussion

This paper has presented a formal logic of occurrence, to capture the relationship between histories and envisionments and to provide the ability to reason about the consequences of behavioral assumptions. It should be applicable to any system of qualitative physics, since it makes very few assumptions about the details of the underlying physics.

While the specific algorithms described above require the ability to explicitly construct envisionments, these ideas can be used without explicit envisionments. One possibility is to generate a subset of the possible behaviors, using these laws to guide backward chaining. Another is to prove general statements about classes of physical behavior, in the style of argument used in classical thermodynamics. We are exploring both styles of reasoning.

We are also applying these algorithms to envisionments involving Qualitative Process theory. The implementation is taking the form of a post-processor to QPE, our new QP implementation which generates total envisionments. The major unsolved problem is automatically detecting transverse quantities. One avenue is to simply declare them — energy, after all, is the typical choice. Another possibility is to extend Weld's *aggregation* technique [12] to extract transverse quantities as a side-effect of the cycle summarization procedure.

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